USING NEURAL NETWORKS FOR THE IDENTIFICATION OF A CLASS OF HYBRID DYNAMIC SYSTEMS

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Abstract: This paper addresses the problem of the identification of Hybrid Dynamic System (HDS) by focusing the attention on the identification of a global model that predicts the continuous outputs of the HDS. The proposed approach considers the identification of HDS in terms of the architectures and the learning algorithms developed for Feed-Forward neural networks.

Keywords: Neural Network, Hybrid Systems, Identification.

1. INTRODUCTION

Over the past years, the study of Hybrid Dynamic systems (HDS), which combines continuous and discrete dynamics, has attracted increased attention. Most works that deal with the control, the analysis and the diagnosis of hybrid systems are based on the assumption that the model of the hybrid system is available (Branicky, 1996, Coquempot *et al.,* 2004). Hence, it seems that little attention has been paid to the problem of obtaining a model from a given input-output data generated by a hybrid system (Bemporad *et al.*, 2001; Bemporad *et al.,* 2004, Ferrari-Trecate *et al.*, 2003 Hoffmann and Engell, 1998; Juloski *et al.*, 2004; Simon and Engell, 2001; Vidal *et al.*, 2004).

Early work on the identification of hybrid systems employ statistical methods to detect the discrete change of the continuous dynamics (Hoffmann and Engell, 1998). Based on this work, Simon and Engell (2001) decompose the I/O data obtained by hybrid linear systems on wavelets and characterize the discrete events (i.e. the switching) as maxima in the coefficients of the transform. Once, the switching points are detected, the set of data is partitioned into sections in which a constant set of model parameters is estimated to describe the continuous dynamics.

Most of the proposed approaches for the identification of HDS concern the classes of switched linear and PieceWise Affine system. They can be classified into the Mixed_Integer Programming approach (Bemporad *et al.*, 2001), the clustering-based approach (Ferrari-Trecate *et al.,* 2003), the bounded-error approach (Bemporad *et al.*, 2004), the Bayesian approach (Juloski *et al.*, 2004) and the algebraic approach (Vidal *et al.*, 2004).

In (Bemporad *et al.*, 2001), it is shown that the identification problem can be reformulated for two subclasses of PieceWise Affine systems. These reformulations lead to the proposition of algorithms based on Mixed-Integer Linear or Quadratic Programming, which are guaranteed to converge to a global optimum. However, this approach is computationally affordable only for a few measured data because the complexity of the used algorithms is NP-hard. The four other approaches deal particularly with the class of PieceWise Affine AutoRegressive eXogenous (PWARX) system, i.e., models in which the regressor space is partitioned into polyhedra with affine ARX sub-models for each polyhedron. The basic steps that these approaches perform are: the estimation of the parameters, the classification of the data attributed to each mode and the estimation of the polyhedral regions. However, the clustering-based approach (Ferrari-Trecate *et al.*, 2003), the bounded-error approach (Bemporad *et al.*, 2004) and the Bayesian approach (Juloski *et al.*, 2004) require that the ARX submodels orders are fixed. Furthermore, both of the clustering-based approach and the

Bayesian approach require a priori knowledge of the number of the modes.

Unfortunately, the existing identification algorithms deal only with special linear classes of hybrid models. This is a significant limitation, because, to the best of our knowledge, there is no work addressing the case in which the dynamics can be nonlinear and the number of modes, the model parameters and the switching sequence are unknown.

The aim of this paper is to investigate an alternative method that seems to be promising for handling this more challenging case. However, we will restrict this first study to a class of HDS, which is characterised by continuous inputs, continuous outputs and binary discrete inputs. In the proposed approach, we consider the plant as a nonlinear black-box model and try to capture its behaviour globally. In this context, we will consider Feed-Forward neural networks (NNs) as global parametric models in order to show that the behaviours of the considered class of HDS can be predicted with NNs. This permits to obtain global parametric models of HDS without needing to cluster the data or to know the current mode. To the best of our knowledge, and apart from the few remarks addressed by Ferrai-Trecate and Muselli (2002), no works have addressed the issue of using NNs in the context of HDS identification.

The proposed approach has the advantage of considering the identification problem of HDS in terms of the architectures and the learning algorithms developed for NNs. It can therefore deal with system nonlinearities and can be used to track the behaviours of the HDS without a priori knowledge about the current mode. However, this approach will result in a black-box model with a large number of parameters Furthermore, although the obtained NNs represent average models that can fairly approximate a given HDS, they are not able to predict the behaviours of this system with a similar precision in all the modes. Finally, the obtained NNs are not adapted for some control and analysis problems, but they can be very useful to deal with the model-based diagnosis of HDS (Messai *et al.*, 2006).

In this paper, the Feed-Forward-neural-networks based approach for the identification of HDS is presented in Section 2 and, then, this approach is illustrated with the help of a benchmark example in Section 3.

2. IDENTIFICATION OF THE PARAMETERS OF THE NEURAL NETWORKS

Before the presentation of the identification procedure, let us, firstly, attempt to explain the mechanism by which NNs can learn the behaviours of HDS. In fact, the output of the neural network depends on the number of the hidden neurons and the activation of these hidden neurons. Hence, if the set of the hidden neurons is divided into several groups and if each of these groups of neurons is active in a distinct mode of the HDS, then the neural network will fairly approximate the I/O data of the HDS. Of course, the real mechanism is more complex since several subgroups of hidden neurons can be combined to reproduce other modes of the HDS.

As a simple example that illustrates this idea, consider the non linear hybrid system represented by Figure 1. This system, which switch for the function f_1 to the function f_2 when the input U is equal to 20, could be approximated by a NN involving two groups of hidden neurons (figure 2 and 3). The first group is composed of three hidden neuron, which are active for all the values of the inputs $U\leq 20$, (fig. 2). Therefore, the sum of their outputs will reproduce the function f_1 when $U\leq 20$ and this sum will be equal to 1 when $U>20$. On the other hand, the three hidden neurons of the second group are saturated when $U\leq 20$ and the sum of their three outputs will reproduce the function f_2 when $U>20$ (fig. 3). Consequently, the output of a NN with an output neuron using a linear activation function will exactly reproduce the HS if $w_{i,i\in\{1,2,..,6\}} = 1$ and $b = -1$, where $w_{i,i\in\{1,2,..,6\}}$ are the weights between the hidden neurons and the output neuron and b is the bias of the output neuron.

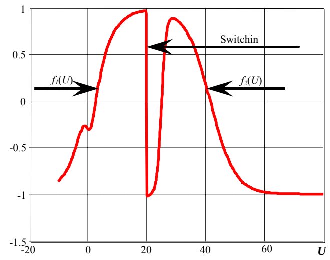

Figure 1: the HDS to be reproduced by the NN

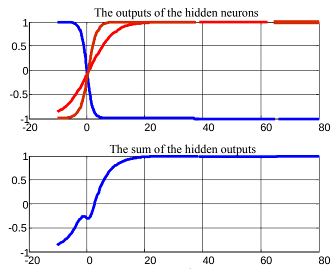

Figure 2: the outputs of the 1st group of hidden neurons

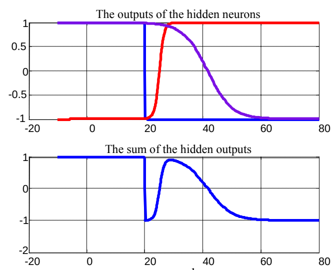

Figure 3: the outputs of the 2nd group of hidden neurons

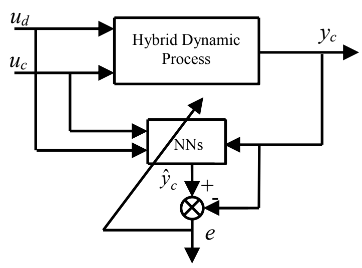

Figure 4: the proposed approach

The proposed identification approach is depicted in figure 4 where the hybrid process is characterised by: the binary discrete inputs (u_d), the continuous inputs (u_c) and the continuous outputs (y_c). The NNs predict the continuous outputs in terms of the past measured input/output variables and without using the state variables. Furthermore, neither the information about the current mode nor the number of the modes are needed to work out the NNs. This configuration is of practical interest, science accurate models of complex hybrid system are often difficult to obtain and only data measured by both continuous and binary sensors are available.

Feed-forward neural networks, with a hidden layer using sigmoidal activation function and an output layer with a linear one, can be used to extract powerful models from experimental data (Carotenuto et al., 1998, Cybenko, 1989). Therefore, the NNs that will be used to predict the continuous outputs y_c are formed by:

- An output layer, containing linear neurons, that provide the continuous outputs,
- A hidden layer containing neurones with sigmoidal activation function,
- An input layer that receives the data from the sensors of the real world hybrid systems.

To begin with the identification procedure, suppose that the input/output data is observed with a sampling period T and that the state changes of the binary inputs are only taken into account at the sampling instants. Then, consider:

$u_c^k = [u_c(1), u_c(2), u_c(3) \ldots u_c(k)]$,

$u_d^k = [u_d(1), u_d(2), u_d(3) \ldots u_d(k)]$,

$y_c^k = [y_c(1), y_c(2), y_c(3) \ldots y_c(k)]$,

where u_c^k, u_d^k and y_c^k represent, respectively, the continuous inputs, the discrete inputs and the continuous outputs at instants kT; $k \in \{1,2,\ldots\}$. The dimensions of the vectors $u_c(k)$, $u_d(k)$ and $y_c(k)$ are respectively given by the number of the continuous inputs, the number of the discrete inputs and the number of the continuous outputs.

In order to model a HDS by NNs, which predict the continuous outputs, we propose to write the relation between $(u_c^{k-1}, u_d^{k-1}, y_c^{k-1})$ and $y_c(k)$ in the form:

$$y_c(k) = g(u_c^{k-1}, u_d^{k-1}, y_c^{k-1}) + e(k) \qquad (1)$$

where g is an unknown function and e is an additive term, indicating that $y(k)$ can not be exactly determined from the previous observations.

Although, the equation (1) can be used to model the HDS, it remains very general to be exploitable and the function $g(u_c^{k-1}, u_d^{k-1}, y_c^{k-1})$ will therefore be decomposed into two functions ϕ and h:

$$\varphi^k = \phi(u_c^{k-1}, u_d^{k-1}, y_c^{k-1}) \qquad (2)$$

$$y_c(k) = h(\varphi^k, \theta) + e(k) \qquad (3)$$

where φ^k is the regressors vector, θ is the parameters vector to be identified and h is a function that expresses the relation between the regressors and the outputs of the NNs. Note that in the case of feed Feed-Forward neural networks, the function h is decomposed into a set of sigmoidal function f representing the neurone of the hidden layer.

At this stage, the identification problem of the HDS, will be a problem of choosing the optimal structure of the NNs. This problem involves the choice of the inputs (i.e., the regression vector), the number of the hidden neurones and the number of the outputs neurones.

Concerning the choice of the number of the output neurons, two alternatives are possible. The first one is to build a single NN with a number of output neurons equal to the number of the continuous outputs of the HDS to be identified. Each output neuron of this NN corresponds to a continuous output of the HDS. The second alternative is to associate a NN with each of the continuous outputs of the system. In this case the number of NNs will be equal to the number of the continuous outputs of the HDS to be identified and each NN will predict one of the continuous outputs.

To choose the regressors, φ^k is decomposed into two parts: the regressors of the continuous measured and/or estimated variables, φ_c^k, and the regressors of the binary discrete inputs, φ_d^k.

To select φ_d^k, we propose to associate a large number of input neurones with each discrete input and to use a pruning algorithm, such as the optimal Brain Surgeon (OBS) algorithm (Reed, 1993), to remove the parameters, and subsequently the input neurons, that are not needed. Hence, if we have N_d discrete inputs, each of which is associated with n_{ed} neurones, the initial regressors φ_d^k will be composed of $n_{ed}.N_d$ elements. These elements represent the values of all the discrete inputs between the instant $(k\text{-}1)T$ and the instant $(k\text{-}n_{ed})T$. Then, the initial regressors φ_d^k will be optimised at the end of the training by the pruning algorithm.

On the other hand, the determination of φ_c^k is derived from the general linear model given by (Chen *et al.*, 1990, Ljung, 1987):

$$A(q^{-1}) y_c(k) = q^{-n_k} \frac{B(q^{-1})}{F(q^{-1})} u_c(k) + \frac{C(q^{-1})}{D(q^{-1})} e(k) \quad (4)$$

where n_k is the delay and the polynomials $A(q^{-1})$, $B(q^{-1}), C(q^{-1}), D(q^{-1})$ and $F(q^{-1})$ are respectively characterised by the orders n_a, n_b, n_c, n_d and n_f. The identification procedure starts with a regressor vector composed of a large identical number of delayed inputs and delayed outputs. The delay n_k is then estimated by modelling the system for various values of n_k and by choosing the value which will correspond to the model providing the smallest residual criterion.

Finally, the regression vector is obtained by: i) modelling the system with ascending values of the orders of the polynomials used in the equation 4, and ii) choosing the model providing the smallest value of the final prediction error as suggested by Akaike (Ljung, 1987):

$$V_{FPE} = \frac{N+d}{N-d} \sum_{k=1}^{N} (e(k))^2 \qquad (5)$$

where, N is the number of data in the training data set and d is the number of weights and the bias (i.e., the parameters) of the NN (Fig. 5).

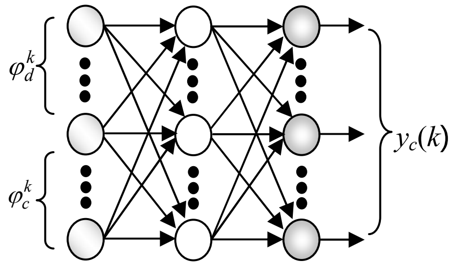

Figure 5: structure of the NN model

3. APPLICATION EXAMPLE

In order to illustrate the proposed approach let us consider the two tanks flow system depicted in the figure 6. This system is the benchmark of the Specific Action on the diagnosis of hybrid systems (AS193) of CNRS[1] and GDR MACS[2]. Although, this system can be perfectly described by a set of mathematical equations, we will consider it as a black-box system equipped with some sensors that provide the I/O data. Consequently, the behaviour of this system will be firstly simulated using a mathematical model of the system. Then, the data obtained in simulation are considered as a set of I/O data measured by fictive sensors and provided to the NNs that predict the next continuous outputs.

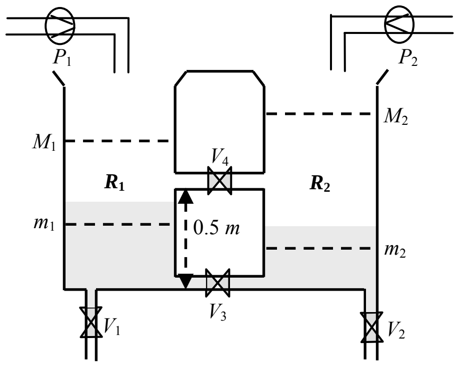

Figure 6: the two tanks system

3.1 Description of the system

The system consists of two cylindrical tanks, R_1 and R_2. Each tank has an input pipe and an output pipe. The input pipes use built-in identical on-off pumps (P_1 and P_2) and the output pipes are controlled by two on-off electro valves, V_1 and V_2. The tanks are connected to each other by means of two pipes, located at the bottom of the tanks and at 0.5 m height. These pipes are also controlled by on-off electro valves, V_3 and V_4. Finally, each tank is equipped with an analogue sensor that measures the level of the fluid.

The system can be represented as a HDS with two continuous variables, h_1 and h_2, representing the height of the fluid in the tanks and six binary inputs: P_1, P_2, V_1, V_2, V_3 and V_4.

For the purpose of simulation let us consider that the dynamics of the pumps are very fast. Hence, we can suppose that the input flows are constants when the pumps are on and are null when the pumps are off.

According to the Torricelli model, the dynamics of the system can be described by the following equations:

$$Q_{P1} = D.P_1, \qquad P_1 \in \{0,1\} \qquad (6)$$

$$Q_{P2} = D.P_2, \qquad P_2 \in \{0,1\} \qquad (7)$$

$$Q_1 = A\sqrt{2.g.h_1}V_1, \qquad V_1 \in \{0,1\} \qquad (8)$$

$$Q_2 = A.\sqrt{2.g.h_2}V_2, \qquad V_2 \in \{0,1\} \qquad (9)$$

$$Q_3 = .\alpha.\sqrt{|h_1 - h_2|}V_3, \qquad V_3 \in \{0,1\} \qquad (10)$$

$$Q_4 = \alpha.\sqrt{|\sup(h_1, 0.5) - \sup(h_2, 0.5|}V_4, \qquad V_4 \in \{0,1\} \qquad (11)$$

$$S\dot{h}_1 = Q_1 - Q_3 - Q_4 \qquad (12)$$

$$S\dot{h}_2 = -Q_2 + Q_3 + Q_4 \qquad (13)$$

where $\alpha = A.\sqrt{2.g}sign(h_1 - h_2)$, D is the constant input flow of both pumps (D=$10^{-4}m^3/Sec$), $Q_{Pi,i\in\{1,2\}}$ is the input flow of tank i, $Q_{i,i\in\{1,2\}}$ is the output flow of tank i, Q_3 is the flow in the pipe C_1 and Q_4 is the flow in the pipe C_2.

In order to avoid either the draining or the overflow of the tanks, the electro valves V_1, V_3 and V_4 as well as the pump P_1 are driven by an algorithm which guarantees the following levels of the fluids in the tanks R_1 and R_2:

$$m_1 \le h_1 \le M_1 \qquad (14)$$

$$m_2 \le h_2 \le M_2 \qquad (15)$$

with M_1=0.6m, M_2=0.75m, m_1=0.4m and m_2=0.2m.

Finally, to further complicate the modelling tasks, the pump P_2 and the electro valve V_2 are considered as perturbations which cannot be controlled. Hence, P_2 and V_2 were opened and closed according to two Nearly Random Binary Sequences of length: $l_{p2} \in [10,30]$ and $l_{v2} \in [30,50]$.

Although the number of modes of this benchmark example is known (64 modes), the identification approach is designed to deal with unknown number of modes. Hence, the knowledge about the number of modes and the current mode will not be used to build the NNs.

3.2 The NN identification results

The above hybrid system is modelled by two feed-forward neural networks according to the approach presented in section 2. Each NN predicts the level of the liquid in the corresponding tank.

[1] Centre National de la Recherche Scientifique
[2] Groupement de Recherche "Modélisation, Analyse et Conduite des Systèmes dynamiques" of CNRS

During the simulations phase, two data sets, each containing $N = 5000$ data, were generated. The first data set, called the identification data set, is used to find the optimal structure of the NNs (i.e., the number of neurons in each layer and the parameters of each NN) and the second data set, called the validation data set, is used to verify the accuracy of the obtained structures when other data are used.

These data sets that have been used to build the NNs were obtained by:

- Fixing the sampling period to T=1 *Sec*,
- Simulating the behaviours of the system according to the equations 6 to 13. The initial conditions were: $h_0 = h_1 = 0\,m$ for the identification data set and $h_0 = h_1 = 1\,m$ for the validation data sets,
- Adding Gaussian noises with zero mean and a standard deviation $\sigma = 0.01$ to the data sets. These noises were added to evaluate the accuracy of the obtained model in the presence of noise.

In order to obtain the structure of the NNs, 5 input neurons were associated with each binary input at the beginning of the identification procedure. Consequently the initial vector φ_d^k was composed by 30 elements that represent the 5 last states of the valves and the last 5 states of the pumps. Note that this initial φ_d^k is considered sufficient because we have observed that the OBS algorithm eliminates some inputs at the end of each training.

The identification procedure is initialised with a regressor vector, φ_c^k, composed of a sufficiently high number of delayed outputs (n_a=5) in order to determine the delay n_k. Once this delay is obtained, the regressors are selected by modelling the system with ascending values of the orders and by choosing the orders of the model providing the smallest value of the final prediction error.

During the identification phase, the parameters of the networks were estimated by means of Levenberg-Marquardt algorithm (Declerq and Dekeyser, 1995) with 20 different initializations and a maximum of *5000* iterations; these several trials were performed in order to select the model with the best performances. Then, starting from a structure with 12 hidden neurons, the Optimal Brain Surgeon (OBS) algorithm (Reed, 1993) was used to remove the parts that are not needed. Hence, after several simulations, according to the approach described in section two, we have opted for two NNs with:

- 21 input neurons that receive the last three outputs of the considered sensor (continuous variables), the last three binary states of the valves and the last three binary state of the pumps,
- 8 hidden neurons,
- 1 output neuron using a linear activation function. The outputs of this neuron correspond to the predicted liquid level in the considered tank.

Figures 7a, 7b, 7c, 8a, 8b and 8c present, respectively, the identification and the validation results of the selected models. The upper part of figures 7a and 8a depicts the predicted and the measured liquid levels. The analysis of these figures shows that it is difficult to distinguish between these levels because the liquid levels predicted by both the NNs closely match the measured data. The lower parts of figures 7a and 7b show that the residuals, which represent the differences between the measured and the predicted levels, are very small and indicate that the relative errors do not exceed 5% of the measured levels. The same remarks apply to figures 8a and 8b. These results validate the identified models and indicate that the NNs are able to estimate the level of the liquid for all the modes of the considered systems.

Other results, not presented here, indicated that the values of the cross correlation functions between the inputs and the outputs are lower than the practical threshold given by Landau (2001). These results pointed out the independence between the residuals and the inputs and show that the NNs are able to reproduce all the dynamics of the system. Furthermore, the analysis of the autocorrelation functions of the residuals has shown that the values of these functions belong to the 99% confidence intervals. These results confirm the independence between the residues and indicate that the residuals can be considered as a white noise. Finally, figures 7c and 8c confirm the analysis of the autocorrelation plots and indicate that the distributions of the residuals are similar to the distributions of the noise that was added to the data sets.

4. CONCLUSION

A methodology to build black-box models of HDS has been proposed. According to this methodology, the behaviours of HDS can be predicted by feed-forward neural networks that track all the modes of the system and the determination of the structure of these neural networks can be viewed as a system identification problem. This approach was illustrated with a simulation example and the obtained results provide strong evidence of the good performances of the obtained model. Several problems remain open, such as the proposition of residual criteria and validation method, which guarantee the same validity of the NNs for all the modes of the HDS.

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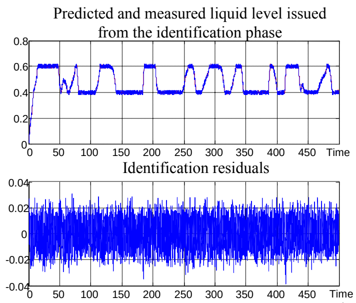

Figure 7a: identification results for the 1st NN

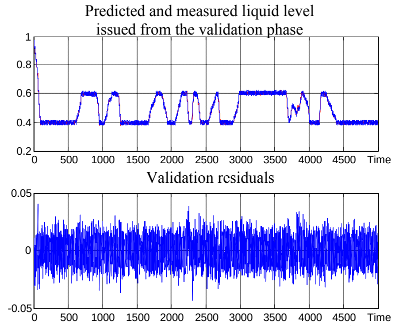

Figure 7b: validation results for the 1st NN

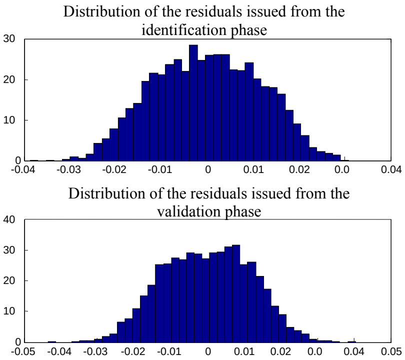

Figure 7c: distribution of the residuals for the 1st NN

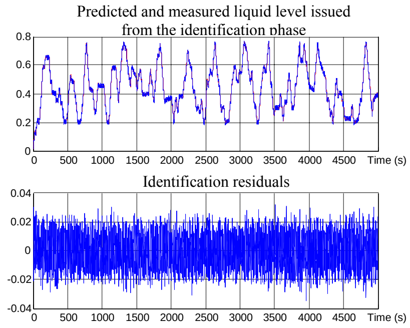

Figure 8a: identification results for the 2nd NN

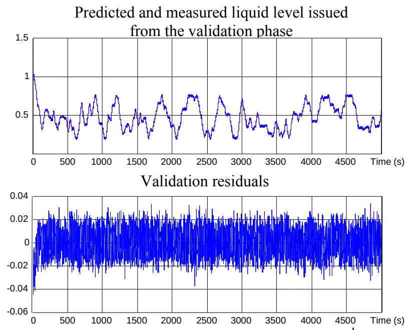

Figure 8b: validation results for the 2nd NN

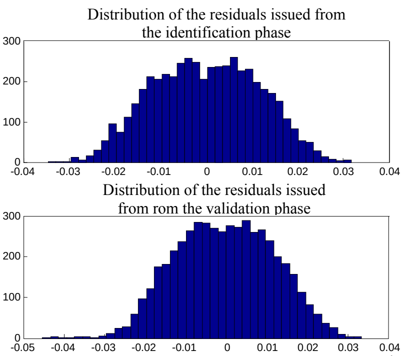

Figure 8c: distribution of the residuals for the 2nd NN