Analysis and Control of Cyber-Physical Systems

Homework 1 — 13 March 2025

Problem 1. Consider the DFA G on alphabet $E = \{a, b, c\}$ with initial state x_0, set of final states $X_m = \{x_1\}$ and transition function

δ	a	b	c
x_0	x_1	$-$	$-$
x_1	$-$	x_2	$-$
x_2	$-$	x_1	x_3
x_3	$-$	$-$	$-$

(a) Give a graphical representation of G.

(b) Discuss if the states of G are: reachable, co-reachable, blocking, dead.

(c) Discuss if G is: reachable, co-reachable, blocking, trim, reversible, complete.

(d) If G is blocking, determine the DFA $trim(G)$, showing both the algebraic and graphical representation.

(e) Determine the language $L(G)$ generated by G and the language $L_m(G)$ accepted by G.

(f) Determine, if they exists, a DFA G' generating $L_m(G)$ and a DFA G'' accepting $L(G)$.

Problem 2. For each of the following languages, determine a trim DFA that accepts it.

(a) Set of words on alphabet $E_1 = \{a, b\}$ where each a is immediately followed by at least two b's.

(b) Set of words on alphabet $E_2 = \{0, 1\}$ not containing substring 000.

(c) The Dyck language on alphabet $E_3 = \{\ (\ ,\ )\ \}$, i.e., set of strings of balanced parenthesis: (), ()(), (()), (()()), ...

Problem 3. The *wolf, goat, and cabbage problem* is a classic river-crossing puzzle in logic and problem-solving, first described by the medieval scholar Alcuin of York (c. 735–804).

A farmer with a wolf, a goat, and a cabbage are on the left bank of a river and must cross to the right bank by boat. The boat can carry only the farmer and a single item. If left unattended together, the wolf would eat the goat, or the goat would eat the cabbage.

Formalize this problem as a discrete event system and describe it by a DFA, whose final state denotes to the completion of the crossing.

You may want to use the following notation to describe the states:

- (F, W, G, C): farmer. wolf, goat and cabbage are on the left bank;
- $(\overline{F}, W, G, C)$: farmer is on the right bank and wolf, goat and cabbage are on the left bank; etc.

and the events:

- f: farmer crossing alone; w (resp. g, c): farmer crossing with wolf (resp., goat, cabbage).

(a) Choose a suitable set of final states and mark in red the forbidden states (goat-cabbage or wolf-goat left unattended).

(b) Determine whether there exists an accepted word that can be generated without passing through any forbidden state and, if so, specify that sequence. If there are several such sequences choose the minimal one: is it unique?

(c) (Bonus 1 point) Consider a version of the puzzle with two goats and a wolf. Discuss if this version has a solution without constructing the corresponding DFA. Please check with ChatGPT if your discussion is correct.