Analysis and Control of Cyber-Physical Systems

Midterm exam — 19 April 2024

Problem 1. (15 points) Consider a nondeterministic finite automaton G on alphabet $E = \{a, b, c\}$ with initial state x_0, set of final states $X_m = \{x_2\}$ and transition relation

$$\Delta = \{(x_0, a, x_0), (x_0, a, x_1), (x_0, b, x_0), (x_1, a, x_2), (x_1, b, x_2), (x_1, c, x_1), (x_2, c, x_2)\}.$$

(a) *(2 points)* Determine the graphical representation of this NFA and discuss which structural elements are nondeterministic.

(b) *(2 points)* Determine if the following words belong to the generated language $L(G)$ and to the marked language $L_m(G)$. You must write all runs that generate these words if applicable.

$$w_1 = aaa; \qquad w_2 = aba; \qquad w_3 = baccc.$$

(c) *(2 points)* Determine the strongly connected components of the NFA. Based on this analysis discuss if it is reversible or blocking.

(d) *(3 points) [This is a general theoretical question]*
What is a DFA G' equivalent a given NFA G? Discuss which what is the upper bound on the number of states of G'. Can this upper bound be reached?

(e) *(4 points)* Build a DFA G' equivalent to G, clearly indicating all the steps followed during the conversion procedure. Which is the cardinality of the state space of this DFA?

(f) *(2 points)* Which are the states reachable in G' by runs that generate strings w_1, w_2, w_3 mentioned above? Discuss how this result is consistent with the analysis you carried out in step (b).

Problem 2. (6 points) Consider two languages $L_1 = \{(ab)^n b \mid n \geq 0\}$ and $L_2 = \{ab^n \mid n \geq 2\}$ on alphabet $E = \{a, b\}$.

(a) *(3 points)* Determine two DFA G_1 and G_2 that accept, respectively, L_1 and L_2.

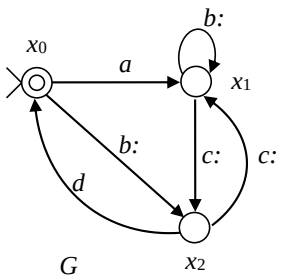
(b) *(3 points)* Determine an automaton G_3 (deterministic or nondeterministic) that accepts $L_1 \cup L_2$.

Problem 3. (9 points) Consider a plant G on alphabet $E = \{a, b, c, d\}$ as in the figure below and the language specification represented by automaton H. The set of controllable events is $E_c = b, c$.

(a) *(4 points)* Discuss whether this specification is controllable.

(b) *(3 points)* Determine a maximally permissive supervisor for plant G capable of enforcing the given specification.

(c) *(2 points)* Discuss if the supervisor you have designed is blocking or not. If blocking, determine a maximally-permissive non-blocking supervisor.



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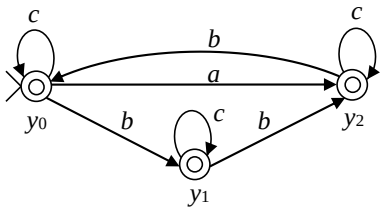

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