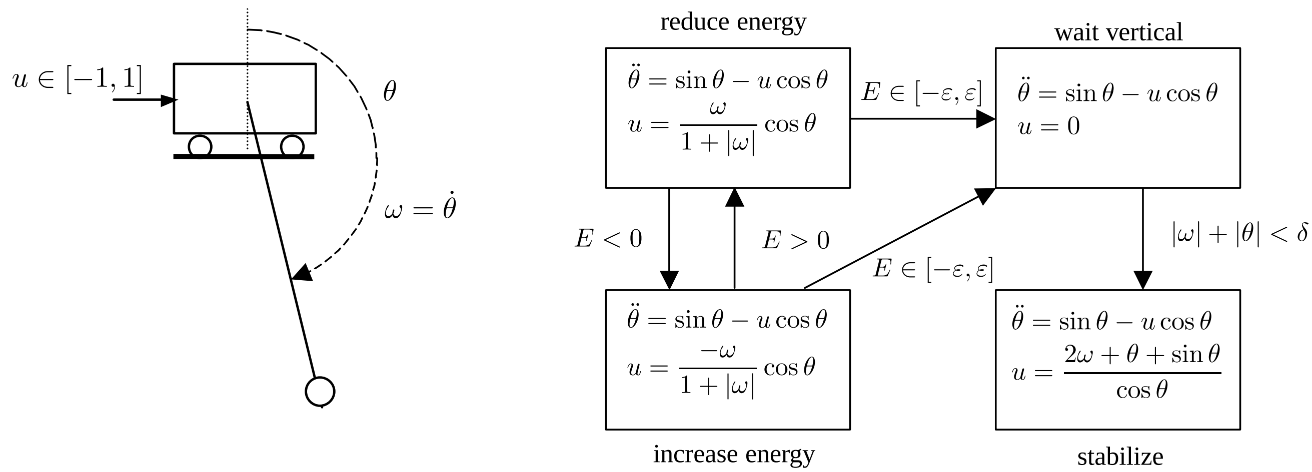
Analysis and Control of Cyber-Physical Systems

Homework 4 — 22 April 2024

Problem 1. Consider the hybrid control system for the stabilization of the inverted pendulum, as shown in the figure. The total energy of the system is $E = \frac{1}{2}\omega^2 + (\cos\theta - 1)$.



(a) Model this system as a hybrid automaton explicitly writing its algebraic representation.

(b) Is this system autonomous? Is this system deterministic? If it not deterministic, suitably modify the invariants to make it deterministic.

(c) What are admissible values of parameter δ? Choose one such value.

(d) What are admissible values of parameter ε? Choose one such value.

(e) Assuming a state $x(t) = (\theta(t), \omega(t))$, simulate the evolution of this system from the following different initial conditions: $x_{0,a} = (0, 2)$, $x_{0,b} = (2, 0)$.

(f) Print the following plots: the state space trajectory in the (x_1, x_2) subspace, the evolution of all components of the continuous state and of the continuous input in time, the evolution of the discrete state in time.

(g) Show that the proposed control scheme cannot stabilize the pendulum starting from from initial condition $\theta(0) = \pi, \omega(0) = 0$. Modify the automaton to also handle this case and show a simulation of its evolution.

Problem 2. Modify the model of the bouncing ball assuming that when the ball reaches the ground the change of velocity $v = -\alpha v^-$ occurs linearly in a fixed time Δ.

(a) Model this system as an autonomous hybrid automaton. You should give both the algebraic and graphical representation of the automaton.

(b) Discuss if this system is zeno.

(c) Simulate the evolution of this system from the initial condition $(h_0, 0)$ with $h_0 = 1\text{m}$ assuming $\alpha = 0.8$ and $\Delta = 0.1\text{s}$. Describe this evolution during the first four bounces by means of a hybrid temporal trajectory and hybrid signals.

(d) Print the following plots: the state space trajectory, the evolution of all components of the continuous state in time, the evolution of the discrete state in time.