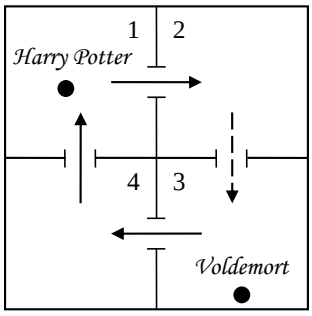
Analysis and Control of Cyber-Physical Systems

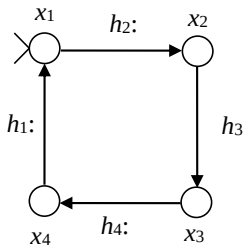
Homework 3 — 4 April 2024

Problem 1. The Marauder's Map in figure (a) below shows Harry Potter and He-Who-Must-Not-Be-Named (V...) in a wing of Hogwarts Castle. They move from room to room passing through one-way doors as indicated on the map. Hermione observes the map and, to prevent the two wizards from meeting, whenever they are in two adjacent rooms she blocks the communication door between the rooms. The door from room 2 to room 3 cannot be blocked.

(a)



(b)



H

The automaton representing Harry is shown in figure (b) above. State x_i denotes that Harry is in room i, while event h_i denotes he is entering room i (for $i = 1, \ldots, 4$). There are no marked states because in this example it is not necessary to distinguish between final and non-final states.

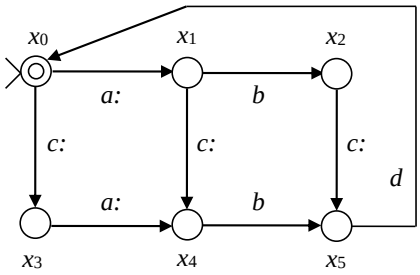
(a) Construct automaton V representing He-Who-Must-Not-Be-Named, in which event v_i denotes he is entering room i (for $i = 1, \ldots, 4$) and whose initial state is consistent with the initial position shown on the map.

(b) Construct the concurrent composition of H and V, indicating as forbidden states all those in which the two characters are in the same room.

(c) Find a maximally permisse control policy to avoid reaching forbidden states. Under this policy, are the two wizards free to reach every room?

Problem 2. Two machine tools, one working at high temperature and one working at low temperature, are always operated in parallel. In the initial and final state both machines are supposed to be loaded and ready to work. The machine working at high temperature performs two operations: first heating (event a) and then processing (event b). The machine operating at low temperature performs only a processing operation (event c). When both machines have finished their respective operations, they are simultaneously unloaded and reloaded (event d).

The behavior of the system can be described by the deterministic finite automaton shown in the figure, where the controllable events are $E_c = \{a, c\}$.



(a) The following behavior must be enforced: during a working cycle the machine operating at high temperature should never finish processing first. Describe this by means of a dynamic specification involving events b and c.

(b) Determine whether this specification is controllable or not.

(c) Determine a supervisor capable of enforcing the specification described in the previous item. Also, determine the closed-loop system, discussing whether the supervisor is blocking or not.