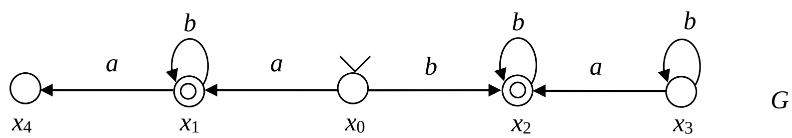
Analysis and Control of Cyber-Physical Systems

Homework 1 — 14 March 2024

Problem 1. Consider the deterministic finite automaton (DFA) shown below.



(a) Determine the algebraic representation of G.

(b) Discuss if the states of G are: reachable, co-reachable, blocking, dead.

(c) Discuss if G is: reachable, co-reachable, blocking, trim, reversible.

(d) Determine the generated language $L(G)$ and the accepted language $L_m(G)$.

(e) Determine the projection $L_m(G) \uparrow \{b\}$. Are there any word $w \in L_m(G) \uparrow \{b\}$ that is not in $L_m(G)$?

(f) If G is not trim, determine $trim(G)$.

Problem 2. For each of the following languages, determine a trim DFA on alphabet $E = \{a, b\}$ that accepts it.

(a) Set of words whose first and third symbol is a.

(b) Set of words where each a is immediately followed by a b and each substring bb is immediately followed by an a.

(c) Set of words w such that $|w|_a = 2 \cdot |w|_b$, i.e., containing twice as many a's than b's.

Problem 3. A classic puzzle, known as *buckets problem*, considers containers with known capacities and aims to reach a specific quantity of liquid in the containers through successive pours. This problem can be usually solved using mathematical reasoning. Here, we aim to show how it can also be solved by brute force, modeling it as a discrete event system and exploring all possible moves.

Suppose there are two containers: C_1 has a capacity of 2 liters and C_2 has a capacity of 3 liters. Initially, both containers are empty. At each step, it is possible to perform one of the following moves indicated by a specific event:

- Event f_i or e_i: fill or empty bucket C_i, for $i \in \{1, 2\}$.
- Event $t_{i,j}$: pour liquid from C_i to C_j until either C_i is empty or C_j is full, for $i, j \in \{1, 2\}$ and $i \neq j$.

Note that the content of the containers after each move will always be an integer, and thus, the overall state of the system can be described by a pair $(x_1, x_2) \in \{0, 1, 2\} \times \{0, 1, 2, 3\}$.

(a) Model this game with a DFA (without final states), showing both its algebraic and graphical representations.

(b) Discuss, from the analysis of the automaton, whether it is possible to reach the following states:

$$i) \quad (0,2); \qquad ii) \quad (1,1); \qquad c) \quad (1,0).$$

For each reachable state, indicate the minimum number of moves required to reach it.

(c) (Bonus 1 point) Present a general analytical solution to the problem (it is ok to google it) and discuss whether the results obtained in the previous point are consistent with this analysis.