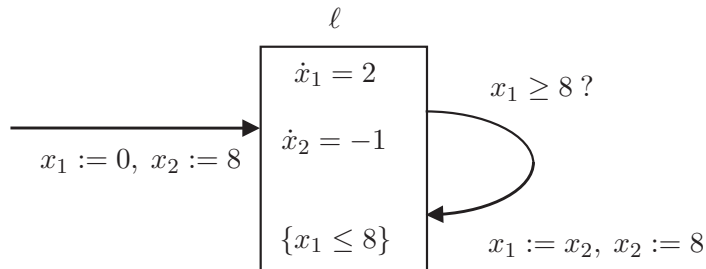


# Analysis and Control of Cyber-Physical Systems

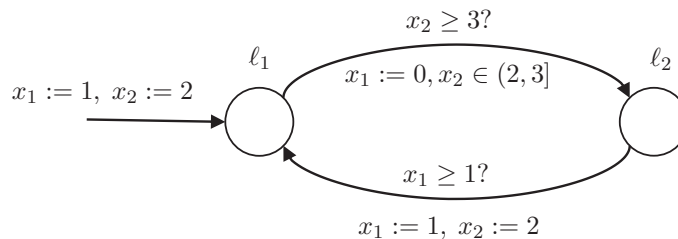
Final exam — 1 June 2023

**Problem 1. [21 pts]** Consider the hybrid automaton  $H$  shown below.



- (3 pts) Determine its algebraic structure and discuss if it belongs to a known subclass of hybrid automata.
- (3 pts) Write a Modelica program to simulate the evolution of the automaton.
- (4 pts) Determine (by hand) the evolution of the automaton during the time interval  $t \in [0, 7]$ . Describe this evolution as hybrid signal, and plot the state variables over time and also the trajectory in the state space.
- (3 pts) Show that the automaton is zeno and compute the time  $T_{zeno}$ .
- (3 pts) Determine a zeno-free hybrid automaton  $H'$  obtained from  $H$  by time-regularization, assuming a dwell time  $\delta = 0.1$ .
- (3 pts) Determine the time-abstract state transition system  $T$  that describes  $H$ .
- (2 pts) Discuss if it is possible to compute the reachability set  $Reach(T)$  using the procedure discussed in class. Could one convert  $H$  into an equivalent timed automata and study reachability using a finite quotient automaton?

**Problem 2. [9 pts]** Consider the timed automaton whose graphical representation is shown below.



- (3 pts) Determine its regions in the continuous state space.
- (3 pts) Construct the region automaton starting from the initial condition.
- (3 pts) Based on the region automaton, what can be said about the reachability of the following states?

- $(\ell_1, (1, 2.5))$ ;
- $(\ell_2, (0.5, 2.6))$ ;
- $(\ell_1, (2, 3.5))$ .