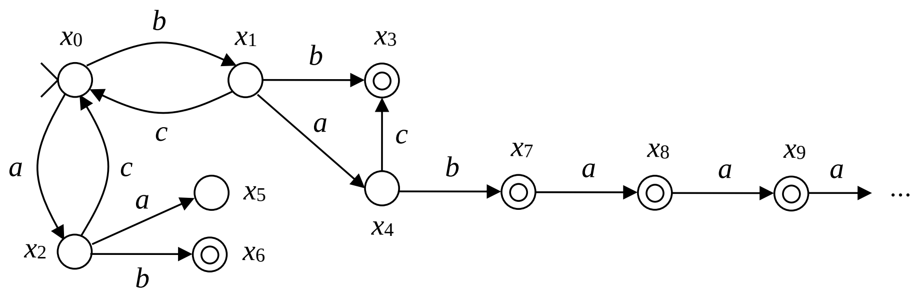
Analysis and Control of Cyber-Physical Systems

Homework 6 — 18 May 2023

Problem 1. Consider the state transition system $T = (S, \Sigma, \rightarrow, S_0, S_F$ shown below, whose state space is countably infinite.



(a) Determine S, Σ, $\rightarrow$, S_0, S_F.

(b) Apply the procedure presented in class for computing the reachability set of T. Show sets R_k and $Reach_K$ for $k = 0, \dots, 6$.

(c) Discuss if the following relation on the state set S of T is an equivalence relation :

$$\mathcal{R} = \{(x_i, x_j) \subseteq S \times S \mid i + j < 3\}.$$

(d) Consider an equivalence relation $\mathcal{R}'$ on the state set of T whose set of equivalence classes corresponds to a partition $\Pi_{\mathcal{R}'} = \{\pi_1, \pi_2, \pi_3\}$, with

$$\pi_1 = \{x_0\}, \qquad \pi_2 = \{x_i \in S \mid 1 \leq i \leq 5\}, \qquad \pi_3 = \{x_i \in S \mid 1 > 5\}.$$

Describe this relation as a subset of $S \times S$ and discuss if it is a bisimulation

(e) Determine a minimal bisimulation $\sim$ over the states of T showing the steps of the procedure you use.

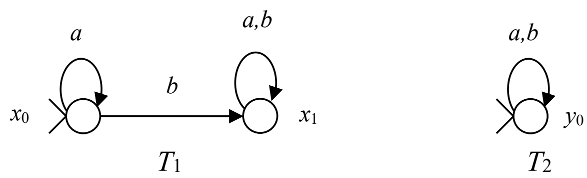
(f) Determine the quotient state transition system $T/\sim$. Are T and $T/\sim$ bisimilar?

(g) Can you find a state transition system that is simulated by $T/\sim$ but is not bisimilar to it?

(h) Discuss if the quotient system can be used to prove/disprove that T is blocking.

(i) Suppose one is interested in determining if a given state (say, x_8) is reachable in a given number of steps (say, k). Can this property be verified by means of the quotient for some k? Can this property be disproved by means of the quotient for some k?

Problem 2. Consider the two state transition systems shown below.



(a) Are they language equivalent?

(b) Does any of the two simulate the other one? Are they bisimilar?

(c) Are they isomorph?

Problem 3. Determine a timed automaton H with external inputs to describe the following system. A lamp is initially "off". When a button is pushed the lamp is switched on with low intensity: in this "low" state if the button is pushed again fast enough (within 2 seconds) the lamp will become brighter else pushing the button will turn the light off. When the light is "bright" if no button is pushed after 3 seconds it goes back to "low" while pushing the button will turn it off.

(a) Determine the regions in the continuous state space X of this automaton.

(b) Determine the number of equivalence classes of the corresponding bisimulation for its time-abstract state transition system and discuss if this number is smaller than or identical to the bound N_S.

(c) Determine the region graph of this automaton.

(d) Can the lamp can be "bright" at total time t = 6? If the region graph does not allow you to verify this, try to answer this question based on your understanding of how this system works.

(e) Discuss if it may be possible to formally answer the previous question constructing a new timed automaton H', obtained from H adding a new clock that is never reset and measures the total time. Is H' more difficult to analyze?