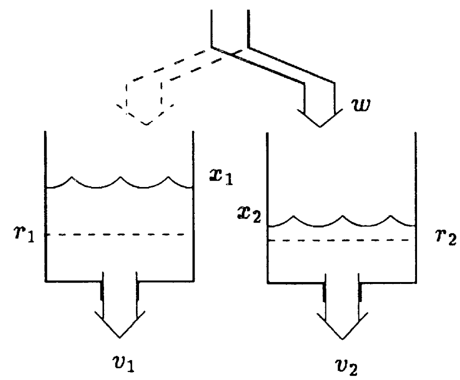
Analysis and Control of Cyber-Physical Systems

Homework 5 — 5 May 2022

Problem 1. The system shown in the picture consists of two tanks S_1 e S_2 and the volume of water they contain at time t is denoted, respectively, $x_1(t)$ and $x_2(t)$. A single hose, producing a constant flow w, feeds the two tanks. The hose can be in two positions: when on the left it feeds the first tank, when on the right it feeds the second one. The time required to move the hose between the two positions is negligible and can be assumed null.



The output flow of tank S_i (for $i = 1, 2$) is denoted v_i and is constant (when the tanks is not empty). The volume of water in tank S_i should not drop below a reference level r_i and the adopted control policy consists in switching the position of the hose to feed it when its volume reaches the reference level.

(a) Determine the graphical and algebraic structure of a hybrid automaton H describing the controlled system assuming the hose is initially feeding the first tank.

(b) Consider, to solve this and following items, these numerical values: $w = 3.5\ m^3/s$, $v_1 = 1; v_2 = 3\ m^3/s$, $r_1 = r_2 = 1\ m^3$, $x_1(0) = 2\ m^3$, $x_2(0) = 2\ m^3$.

Describe the state evolution of its hybrid automaton up to the first four switches of the hose position in terms of hybrid temporal trajectory and hybrid signals. Can a tank become empty?

(c) Show that the automaton is zeno and explain why in plain words. Determine the time instant T_{zeno}.

(d) Apply space regularization with a minimum state variation $\varepsilon = 0.1$ to determine a non-zeno model. Can a tank become empty?

(e) Determine a condition on w, v_1 and v_2 ensuring that the original automaton is non-zeno.

Problem 2. Consider again the hydraulic system discussed in the previous problem, but assume now $w = 4\, m^3/s$.

(a) Determine the time-abstract state transition system T_{H_τ} associated to the hybrid automatonm H computed in the previous problem.

(b) Determine the reachability set $Reach(T)$.

Problem 3. Consider a time-driven system modelled by the differential equation

$$\dot{x}(t) = f(x(t))$$

where $x(t) \in \mathbb{R}$ and the activity function is

$$f(x) = \begin{cases} -2x & \text{if } x < -1 \\ x & \text{if } -1 \leq x < 1 \\ -2x & \text{if } 1 \leq x \end{cases}$$

(a) Draw the plot of the activity function.

(b) Discuss for which values of the initial condition one can ensure the existence of a local solution. Are these solutions unique? Are they global?

(c) Determine the evolution of the system for $t \geq 0$ starting from the initial condition $x(0) = 0.5$, both analytically and via simulation. What happens when the state reaches a point of discontinuity of the activity function?

(d) Determine, starting from the previously given initial condition, the evolution of the system for $t \geq 0$ using a Filippov solution and draw a plot of this signal.

(e) Determine, if possible, a hybrid automaton with continuous activity functions, that can also describe this system.