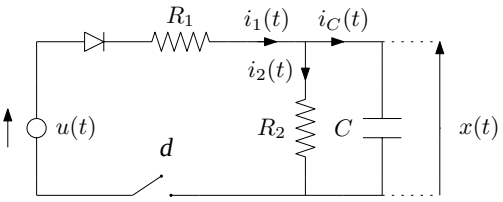
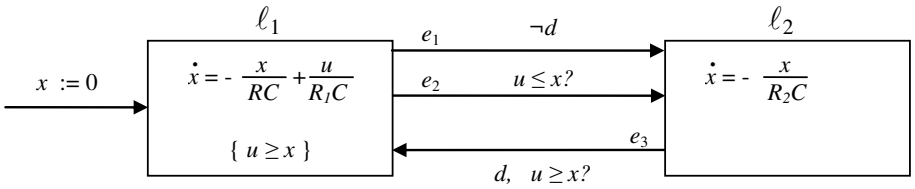
# Analysis and Control of Cyber-Physical Systems

Homework 4 — 27 April 2023

**Problem 1.** Consider the circuit with a diode studied in the Example 10.7 of the class notes, which is shown in the figure below.




The hybrid automaton model of this device is shown in the next figure.

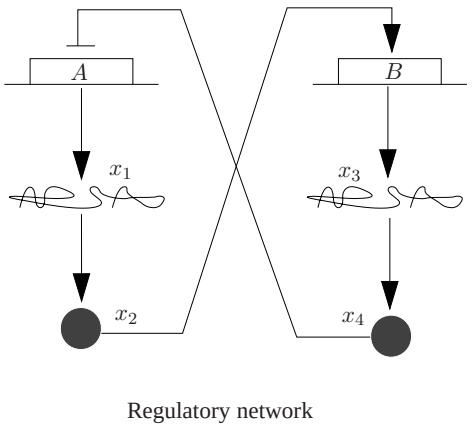



(a) Write an OpenModelica program to model this automaton.

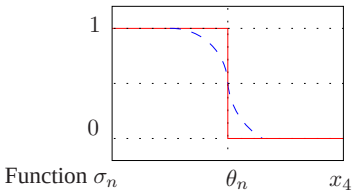
The following values are suggested: $R_1 = 500\ \Omega, \quad R_2 = 700\ \Omega, \quad C = 0.01\ F.$

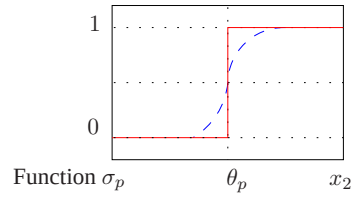
(b) Simulate its evolution for $t \in [0, 40]$ assuming that $u(t) = \sin(t)$ and that the switch is opened for $t \in [20, 30]$ and closed otherwise.

**Problem 2.** Consider the gene regulatory network consisting of two genes $A$ and $B$ and discussed in studied in Section 9.2.5 of the class notes. The network is shown in the figure below on the left.




Regulatory network




Here $x_1, x_3 \in [0, 1]$ and $x_2, x_4 \in [0, 1]$ are the concentrations of the mRNAs and the proteins produced by the genes $A$ and $B$, respectively. The two genes are mutually dependent: the presence of protein $B$ inhibits the expression of gene $A$, while the presence of protein $A$ activates the expression of gene $B$.

The system's dynamics are given by:

$$\begin{cases} \dot{x}_1 &= a_1\sigma_n(x_4) - b_1x_1, \\ \dot{x}_2 &= a_2x_1 - b_2x_2, \\ \dot{x}_3 &= a_3\sigma_p(x_2) - b_3x_3, \\ \dot{x}_4 &= a_4x_3 - b_4x_4, \end{cases}$$

where $a_i, b_i \in \mathbb{R}_{\geq 0}$ $(i = 1, 2, 3, 4)$ are the production and degradation rate constants, and $\sigma_n$ and $\sigma_p$ are the non-linear functions shown in the figure on the right.

We assume $\sigma_n$ and $\sigma_p$ can be approximated by the piecewise-constant functions shown in red in the figure on the right, i.e.,

$$\sigma_n(x_4) = -0.5\text{sign}(x_4 - \theta_n) + 0.5 \quad \text{e} \quad \sigma_p(x_2) = 0.5\text{sign}(x_2 - \theta_p) + 0.5.$$

Consider the following values of the parameters.

| | | | |
|---|---|---|---|
| $a_1 = 1$ | $a_2 = 1$ | $a_3 = 2$ | $a_4 = 1$ |
| $b_1 = 2$ | $b_2 = 3$ | $b_3 = 3$ | $b_4 = 4$ |
| $\theta_n = 0.5$ | $\theta_p = 0.6$ | | |

(a) Model the nework by an hybrid automaton, showing both graphical and algebraic representation.

(b) Discuss whether this model is autonomous or controlled, deterministic or non-deterministic.

(c) Simulate this automaton with OpenModelica, considering the following initial conditions:

$$\begin{aligned} &a) && x_1 = x_3 = 1; \quad x_2 = x_4 = 0 \\ &b) && x_1 = x_4 = 1; \quad x_2 = x_3 = 0 \end{aligned}$$

You must show the following plots: trajectory in subspace $(x_1, x_2)$, time evolution of all continuous state signals, time evolution of the discrete state.

(d) Discuss if the systems reaches an equilibrium condition and how this condition changes for different values of $\theta_p \in [0, 1]$.